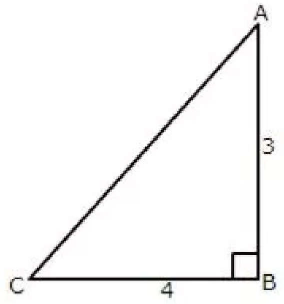
Chapter 22. Trigonometrical Ratios [Sine, Consine, Tangent of an Angle and their Reciprocals]

Exercise 22(A)

Solution 1:

Given angle $ABC = 90^\circ$



$\Rightarrow AC^2 = AB^2 + BC^2$ (AC is hypotenuse)

$\Rightarrow AC^2 = 3^2 + 4^2$

$\therefore\ AC^2 = 9 + 16 = 25$ and $AC = 5$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3}{5}$$

(iii)

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3}{4}$$

(iv)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{BC} = \frac{5}{4}$$

(v)

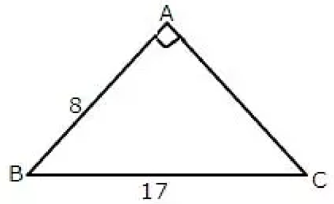
$$\text{cosec}\, C = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB} = \frac{5}{3}$$

(vi)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{3}{4}$$

Solution 2:

Given angle $BAC = 90^\circ$



$\Rightarrow BC^2 = AB^2 + AC^2$ (BC is hypotenuse)

$\Rightarrow 17^2 = 8^2 + AC^2$

$\therefore \quad AC^2 = 289 - 64 = 225$ and $AC = 15$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{AC} = \frac{8}{15}$$

(iii)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\sin^2 B + \cos^2 B = \left(\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2$$
$$= \frac{225+64}{289}$$
$$= \frac{289}{289}$$
$$= 1$$

(iv)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

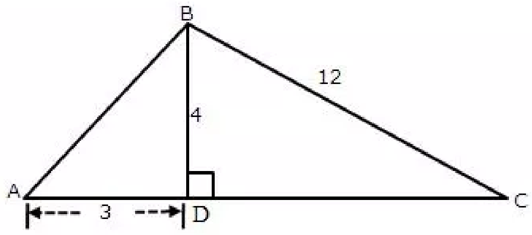
$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\sin B \cdot \cos C + \cos B \cdot \sin C = \frac{15}{17} \cdot \frac{15}{17} + \frac{8}{17} \cdot \frac{8}{17}$$
$$= \frac{225+64}{289}$$
$$= \frac{289}{289}$$
$$= 1$$

Solution 3:

Consider the diagram as



Given angle $ADB = 90^\circ$ and $BDC = 90^\circ$

$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow AB^2 = 3^2 + 4^2$

$\therefore AB^2 = 9 + 16 = 25$ and $AB = 5$

$\Rightarrow BC^2 = BD^2 + DC^2$ (BC is hypotenuse in ΔBDC)

$\Rightarrow DC^2 = 12^2 - 4^2$

$\therefore DC^2 = 144 - 16 = 128$ and $DC = 8\sqrt{2}$

(i)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{3}{5}$$

(ii)

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{5}{4}$$

(iii)

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{BD}{AD} = \frac{4}{3}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{AD} = \frac{5}{3}$$

$$\tan^2 A - \sec^2 A = \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$$

$$= \frac{16}{9} - \frac{25}{9}$$

$$= \frac{-9}{9}$$

$$= -1$$

(iv)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

(v)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{BC}{DC} = \frac{12}{8\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

(vi)

$$\cot C = \frac{\text{base}}{\text{perpendicular}} = \frac{DC}{BD} = \frac{8\sqrt{2}}{4} = 2\sqrt{2}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

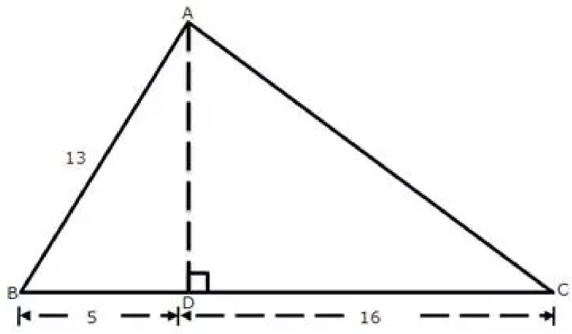
$$\cot^2 C - \frac{1}{\sin^2 C} = \left(2\sqrt{2}\right)^2 - \frac{1}{\left(\frac{1}{3}\right)^2}$$

$$= 8 - 9$$

$$= -1$$

Solution 4:

Given angle $ADB = 90^\circ$ and $ADC = 90^\circ$



$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow 13^2 = AD^2 + 5^2$

$\therefore \quad AD^2 = 169 - 25 = 144$ and $AD = 12$

$\Rightarrow AC^2 = AD^2 + DC^2$ (AC is hypotenuse in ΔADC)

$\Rightarrow AC^2 = 12^2 + 16^2$

$\therefore \quad AC^2 = 144 + 256 = 400$ and $AC = 20$

(i)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{13}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$$

(iii)

$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{13}{5}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{5}$$

$$\sec^2 B - \tan^2 B = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2$$

$$= \frac{169 - 144}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

(iv)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$$

$$\sin^2 C + \cos^2 C = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

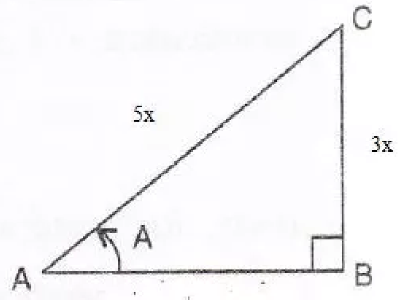
$$= \frac{9+16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

Solution 5:

Consider the diagram below:



$\sin A = \frac{3}{5}$

$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{BC}{AC} = \frac{3}{5}$

Therefore if length of $BC = 3x$, length of $AC = 5x$

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$AB^2 + (3x)^2 = (5x)^2$

$AB^2 = 25x^2 - 9x^2 = 16x^2$

$\therefore AB = 4x(\text{base})$

Now

(i)

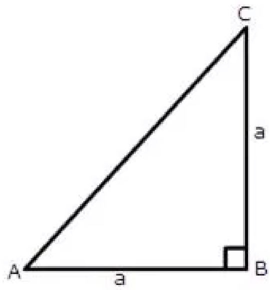
$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3x}{4x} = \frac{3}{4}$

(ii)

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

Solution 6:

Given angle $ABC = 90^\circ$ in the figure



$\Rightarrow AC^2 = AB^2 + BC^2$ (AC is hypotenuse in ΔABC)

$\Rightarrow AC^2 = a^2 + a^2$

$\therefore\ AC^2 = 2a^2$ and $AC = \sqrt{2}a$

Now

(i) $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

(ii) $\sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AC}{AB} = \dfrac{\sqrt{2}a}{a} = \sqrt{2}$

(iii) $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

$\cos A = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

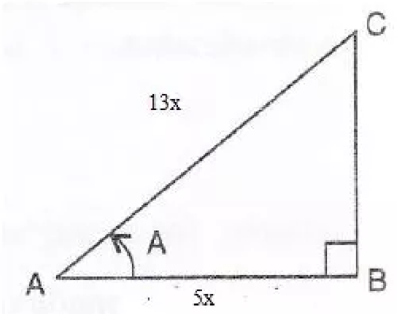
$$\cos^2 A + \sin^2 A = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Solution 7:

Consider the diagram below:



$$\cos A = \frac{5}{13}$$

$$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{5}{13} \Rightarrow \frac{AB}{AC} = \frac{5}{13}$$

Therefore if length of $AB = 5x$, length of $AC = 13x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 + BC^2 = (13x)^2$$

$$BC^2 = 169x^2 - 25x^2 = 144x^2$$

$$\therefore BC = 12x(\text{perpendicular})$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{12x}{5x} = \frac{12}{5}$$

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{12x}{13x} = \frac{12}{13}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{5x}{12x} = \frac{5}{12}$$

(i)

$$\frac{\sin A - \cot A}{2\tan A}$$

$$= \frac{\frac{12}{13} - \frac{5}{12}}{2\left(\frac{12}{5}\right)}$$

$$= \frac{79}{156} \cdot \frac{5}{24}$$

$$= \frac{395}{3744}$$

(ii)

$$\cot A + \frac{1}{\cos A}$$

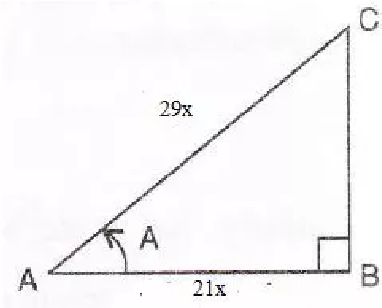
$$= \frac{5}{12} + \frac{1}{\frac{5}{13}}$$

$$= \frac{5}{12} + \frac{13}{5}$$

$$= \frac{181}{60}$$

Solution 8:

Consider the diagram below:



$\sec A = \frac{29}{21}$

$i.e. \frac{\text{hypotenuse}}{\text{base}} = \frac{29}{21} \Rightarrow \frac{AC}{AB} = \frac{29}{21}$

Therefore if length of $AB = 21x$, length of $AC = 29x$

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(21x)^2 + BC^2 = (29x)^2$

$BC^2 = 841x^2 - 441x^2 = 400x^2$

$\therefore BC = 20x(\text{perpendicular})$

Now

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{20x}{29x} = \frac{20}{29}$

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{20x}{21x} = \frac{20}{21}$

Therefore

$\sin A - \frac{1}{\tan A}$

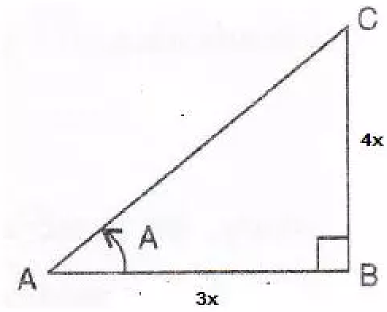
$= \frac{20}{29} - \frac{1}{\frac{20}{21}}$

$= \frac{20}{29} - \frac{21}{20}$

$= -\frac{209}{580}$

Solution 9:

Consider the diagram below:



$\tan A = \frac{4}{3}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$

Therefore if length of $AB = 3x$, length of $BC = 4x$

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x(\text{hypotenuse})$

Now

$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$

$\text{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$

Therefore

$\frac{\text{cosec} A}{\cot A - \sec A}$

$= \frac{\frac{5}{4}}{\frac{3}{4} - \frac{5}{3}}$

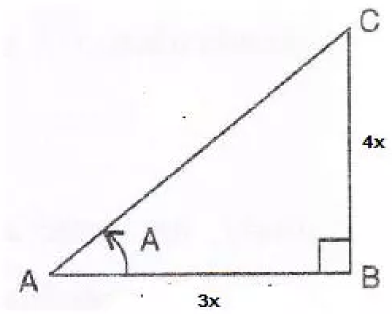
$= \frac{\frac{5}{4}}{\frac{-11}{12}}$

$= -\frac{60}{44}$

$= -\frac{15}{11}$

Solution 10:

Consider the diagram below:



$4\cot A = 3$

$\cot A = \frac{3}{4}$

$i.e.\ \frac{\text{base}}{\text{perpendicular}} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$

Therefore if length of AB = 3x, length of BC = 4x
Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x(\text{hypotenuse})$

(i)

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

(ii)

$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

(iii)

$\text{cosec}\, A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$

$\cot A = \frac{3}{4}$

$\text{cosec}^2 A - \cot^2 A$

$= \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

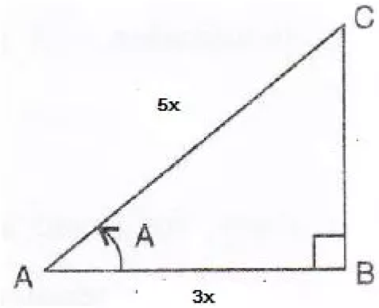
$= \frac{25-9}{16}$

$= \frac{16}{16}$

$= 1$

Solution 11:

Consider the diagram below:



$\cos A = 0.6$

$\cos A = \frac{6}{10} = \frac{3}{5}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$

Therefore if length of AB = 3x, length of AC = 5x

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + BC^2 = (5x)^2$

$BC^2 = 25x^2 - 9x^2 = 16x^2$

$\therefore BC = 4x(\text{perpendicular})$

Now all other trigonometric ratios are

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

$\text{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$

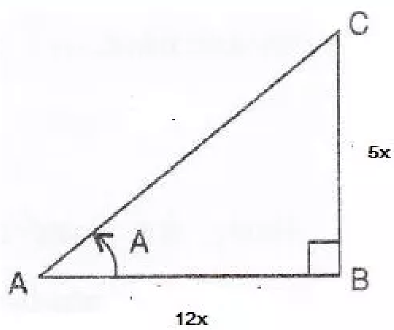
$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{4x}{3x} = \frac{4}{3}$

$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{3x}{4x} = \frac{3}{4}$

Solution 12:

Consider the diagram below:



$\tan A = \frac{5}{12}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{5}{12} \Rightarrow \frac{BC}{AB} = \frac{5}{12}$

Therefore if length of AB = 12x, length of BC = 5x

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(12x)^2 + (5x)^2 = AC^2$

$AC^2 = 144x^2 + 25x^2 = 169x^2$

$\therefore AC = 13x(\text{hypotenuse})$

(i)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$$

(ii)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{5x}{13x} = \frac{5}{13}$$

(iii)

$$\frac{\cos A + \sin A}{\cos A - \sin A}$$

$$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

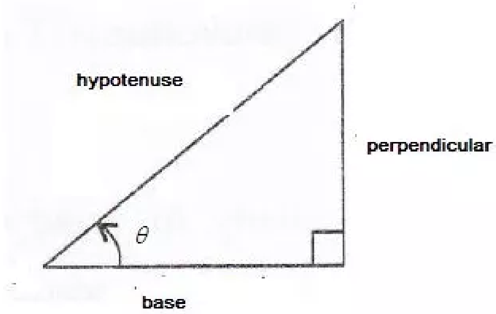
$$= \frac{\frac{17}{13}}{\frac{7}{13}}$$

$$= \frac{17}{7}$$

$$= 2\frac{3}{7}$$

Solution 13:

Consider the diagram below:



$$\sin \theta = \frac{p}{q}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{p}{q}$$

Therefore if length of perpendicular = px, length of hypotenuse = qx

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$$

$$\text{base}^2 + (px)^2 = (qx)^2$$

$$\text{base}^2 = q^2x^2 - p^2x^2 = \left(q^2 - p^2\right)x^2$$

$$\therefore \text{ base} = \sqrt{q^2 - p^2}\,x$$

Now

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{q^2 - p^2}}{q}$$

Therefore

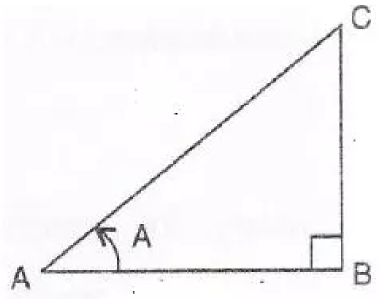
$$\cos \theta + \sin \theta$$

$$= \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q}$$

$$= \frac{p + \sqrt{q^2 - p^2}}{q}$$

Solution 14:

Consider the diagram below:



$$\cos A = \frac{1}{2}$$

$$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$$

Therefore if length of AB = x, length of AC = 2x

Since

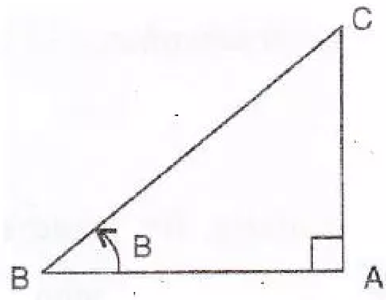
$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + BC^2 = (2x)^2$$

$$BC^2 = 4x^2 - x^2 = 3x^2$$

$$\therefore BC = \sqrt{3}x(\text{perpendicular})$$

Consider the diagram below:



$$\sin B = \frac{1}{\sqrt{2}}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

Therefore if length of AC = x, length of $BC = \sqrt{2}x$

Since

$$AB^2 + AC^2 = BC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$AB^2 + x^2 = (\sqrt{2}x)^2$$

$$AB^2 = 2x^2 - x^2 = x^2$$

$$\therefore AB = x(\text{base})$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$$

Therefore

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

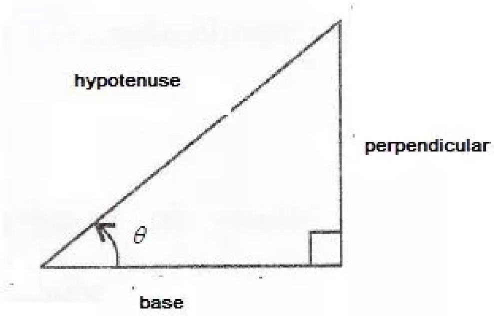
$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

Solution 15:

Consider the diagram below:



$$5\cot\theta = 12$$

$$\cot\theta = \frac{12}{5}$$

$$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{12}{5}$$

Therefore if length of base = 12x, length of perpendicular = 5x

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$$

$$(12x)^2 + (5x)^2 = \text{hypotenuse}^2$$

$$\text{hypotenuse}^2 = 144x^2 + 25x^2 = 169x^2$$

$$\therefore \text{hypotenuse} = 13x$$

Now

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{13x}{5x} = \frac{13}{5}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{13x}{12x} = \frac{13}{12}$$

Therefore

$$\operatorname{cosec}\theta + \sec\theta$$

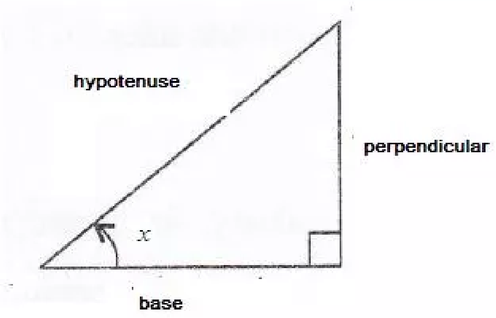
$$= \frac{13}{5} + \frac{13}{12}$$

$$= \frac{221}{60}$$

$$= 3\frac{41}{60}$$

Solution 16:

Consider the diagram below:



$\tan x = 1\frac{1}{3}$

$\tan x = \frac{4}{3}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3}$

Therefore if length of base = 3x, length of perpendicular = 4x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore \text{hypotenuse} = 5x$

Now

$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$

$\cos x = \frac{\text{base}}{\text{hypotenuse}} = \frac{3x}{5x} = \frac{3}{5}$

Therefore

$4\sin^2 x - 3\cos^2 x + 2$

$= 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$

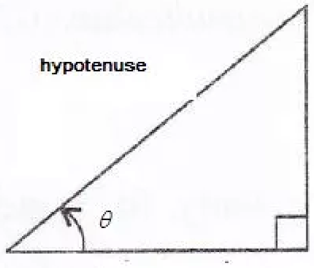
$= \frac{64}{25} - \frac{27}{25} + 2$

$= \frac{87}{25}$

$= 3\frac{12}{25}$

Solution 17:

Consider the diagram below:



$\operatorname{cosec}\theta = \sqrt{5}$

$i.e. \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{\sqrt{5}}{1}$

Therefore if length of hypotenuse $= \sqrt{5}x$, length of perpendicular = x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$ [Using Pythagoras Theorem]

$\text{base}^2 + (x)^2 = (\sqrt{5}x)^2$

$\text{base}^2 = 5x^2 - x^2 = 4x^2$

$\therefore \text{base} = 2x$

Now

$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{x}{\sqrt{5}x} = \dfrac{1}{\sqrt{5}}$

$\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{2x}{\sqrt{5}x} = \dfrac{2}{\sqrt{5}}$

(i)

$2 - \sin^2\theta - \cos^2\theta$

$= 2 - \left(\dfrac{1}{\sqrt{5}}\right)^2 - \left(\dfrac{2}{\sqrt{5}}\right)^2$

$= 2 - \dfrac{1}{5} - \dfrac{4}{5}$

$= \dfrac{5}{5}$

$= 1$

(ii)

$2 + \dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$

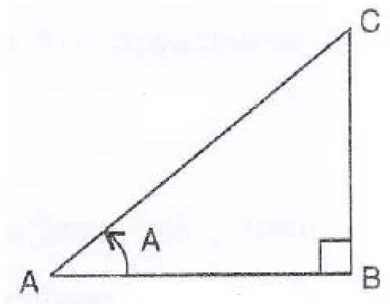
$= 2 + \dfrac{1}{\left(\dfrac{1}{\sqrt{5}}\right)^2} - \dfrac{\left(\dfrac{2}{\sqrt{5}}\right)^2}{\left(\dfrac{1}{\sqrt{5}}\right)^2}$

$= 2 + 5 - 4$

$= 3$

Solution 18:

Consider the diagram below:



$\sec A = \sqrt{2}$

$i.e. \frac{\text{hypotenuse}}{\text{base}} = \frac{\sqrt{2}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$

Therefore if length of AB = x, length of $AC = \sqrt{2}x$

Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(x)^2 + BC^2 = (\sqrt{2}x)^2$

$BC^2 = 2x^2 - x^2 = x^2$

$\therefore BC = x(\text{perpendicular})$

Now

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

Therefore

$\frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A}$

$= \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 5(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$

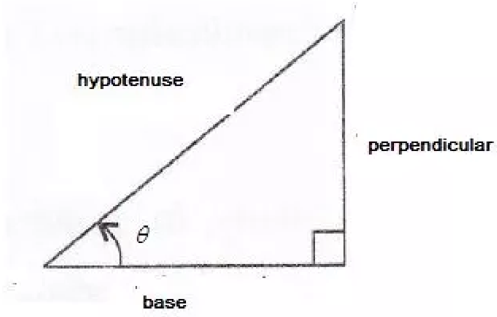
$= \frac{\frac{13}{2}}{\frac{7}{2}}$

$= \frac{13}{7}$

$= 1\frac{6}{7}$

Solution 19:

Consider the diagram below:



$\cot\theta = 1$

$i.e. \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{1}{1}$

Therefore if length of base = x, length of perpendicular = x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$

$(x)^2 + (x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = x^2 + x^2 = 2x^2$

$\therefore \text{hypotenuse} = \sqrt{2}x$

Now

$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{x}{\sqrt{2}x} = \dfrac{1}{\sqrt{2}}$

$\tan\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{x}{x} = 1$

Therefore

$5\tan^2\theta + 2\sin^2\theta - 3$

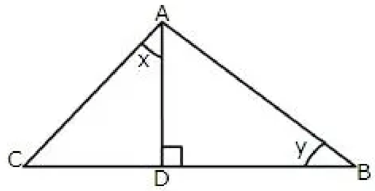
$= 5(1)^2 + 2\left(\dfrac{1}{\sqrt{2}}\right)^2 - 3$

$= 5 + 1 - 3$

$= 3$

Solution 20:

Given angle $DAC = 90^\circ$ and $\angle ADB = 90^\circ$ in the figure



$\Rightarrow AC^2 = AD^2 + DC^2 \ (AC \text{ is hypotenuse in } \Delta ADC)$

$\Rightarrow AD^2 = 26^2 - 10^2$

$\therefore \ AD^2 = 576 \text{ and } AD = 24$

Again

$\Rightarrow AB^2 = AD^2 + BD^2 \ (AB \text{ is hypotenuse in } \Delta ABD)$

$\Rightarrow AB^2 = 24^2 + 32^2$

$\therefore \ AB^2 = 1600 \text{ and } AB = 40$

Now

(i)

$\cot x = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{AD}{CD} = \dfrac{24}{10} = 2.4$

(ii)

$\sin y = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{24}{40} = \dfrac{3}{5}$

$\tan y = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AD}{BD} = \dfrac{24}{32} = \dfrac{3}{4}$

Therefore

$$\frac{1}{\sin^2 y}-\frac{1}{\tan^2 y}$$

$$=\frac{1}{\left(\frac{3}{5}\right)^2}-\frac{1}{\left(\frac{3}{4}\right)^2}$$

$$=\frac{25}{9}-\frac{16}{9}$$

$$=\frac{9}{9}$$

$$=1$$

(iii)

$$\tan y=\frac{\text{perpendicular}}{\text{base}}=\frac{AD}{BD}=\frac{24}{32}=\frac{3}{4}$$

$$\cos x=\frac{\text{base}}{\text{hypotenuse}}=\frac{AD}{AC}=\frac{24}{26}=\frac{12}{13}$$

$$\cos y=\frac{\text{base}}{\text{hypotenuse}}=\frac{BD}{AB}=\frac{32}{40}=\frac{4}{5}$$

Therefore

$$\frac{6}{\cos x}-\frac{5}{\cos y}+8\tan y$$

$$=\frac{6}{\frac{12}{13}}-\frac{5}{\frac{4}{5}}+8\left(\frac{3}{4}\right)$$

$$=\frac{13}{2}-\frac{25}{4}+6$$

$$=\frac{26-25+24}{4}$$

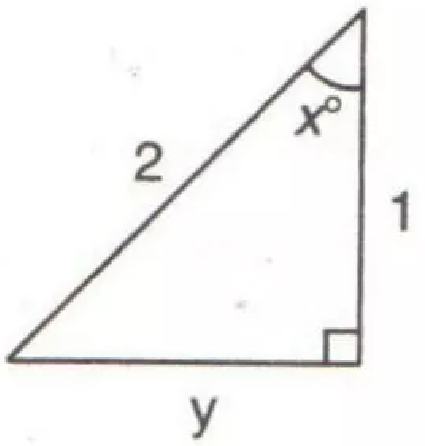
$$=\frac{25}{4}$$

$$=6\frac{1}{4}$$

Exercise 22(B)

Solution 1:

Consider the given figure



(i)

Since the triangle is a right angled triangle, so using Pythagorean Theorem

$2^2 = y^2 + 1^2$

$y^2 = 4 - 1 = 3$

$y = \sqrt{3}$

(ii)

$\sin x^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

(iii)

$\tan x^\circ = \frac{\text{perpendicular}}{\text{base}} = \sqrt{3}$

$\sec x^\circ = \frac{\text{hypotenuse}}{\text{base}} = 2$

Therefore

$(\sec x^\circ - \tan x^\circ)(\sec x^\circ + \tan x^\circ)$

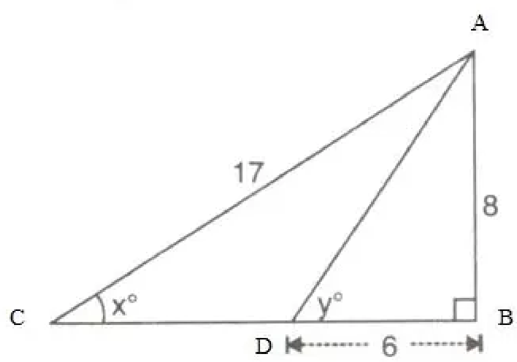
$= (2 - \sqrt{3})(2 + \sqrt{3})$

$= 4 - 3$

$= 1$

Solution 2:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AD^2 = 8^2 + 6^2$

$AD^2 = 64 + 36 = 100$

$AD = 10$

Also

$BC^2 = AC^2 - AB^2$

$BC^2 = 17^2 - 8^2 = 225$

$BC = 15$

(i)

$$\sin x^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{8}{17}$$

(ii)

$$\cos y^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

(iii)

$$\sin y^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AD} = \frac{8}{10} = \frac{4}{5}$$

$$\cos y^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

$$\tan x^\circ = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{8}{15}$$

Therefore

$$3\tan x^\circ - 2\sin y^\circ + 4\cos y^\circ$$

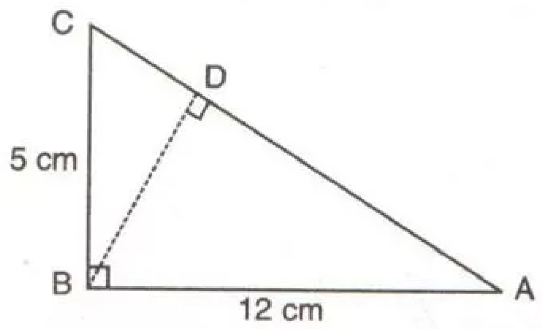
$$= 3\left(\frac{8}{15}\right) - 2\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right)$$

$$= \frac{8}{5} - \frac{8}{5} + \frac{12}{5}$$

$$= 2\frac{2}{5}$$

Solution 3:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AC^2 = 5^2 + 12^2$

$AC^2 = 25 + 144 = 169$

$AC = 13$

In ΔCBD and ΔCBA, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90^\circ$ so therefore $\angle CBD = \angle CAB$.

Therefore ΔCBD and ΔCBA are similar triangles according to AAA Rule

So

$$\frac{AC}{BC} = \frac{AB}{BD}$$

$$\frac{13}{5} = \frac{12}{BD}$$

$$BD = \frac{60}{13}$$

(i)

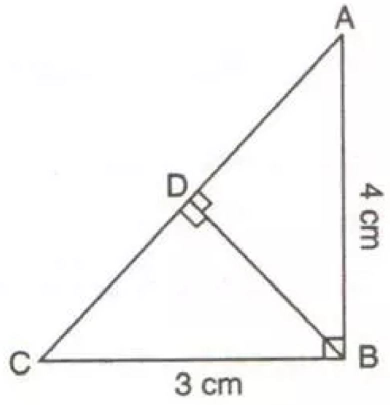
$$\cos\angle DBC = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{\frac{60}{13}}{5} = \frac{12}{13}$$

(ii)

$$\cot\angle DBA = \frac{\text{base}}{\text{perpendicular}} = \frac{BD}{AB} = \frac{\frac{60}{13}}{12} = \frac{5}{13}$$

Solution 4:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9 = 25$

$AC = 5$

In ΔCBD and ΔCBA, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90^\circ$ so therefore $\angle CBD = \angle CAB$.

Therefore ΔCBD and ΔCBA are similar triangles according to AAA Rule

So

$$\frac{AC}{BC} = \frac{AB}{BD}$$

$$\frac{5}{3} = \frac{4}{BD}$$

$$BD = \frac{12}{5}$$

Now using Pythagorean Theorem

$$DC^2 = 3^2 - \left(\frac{12}{5}\right)^2$$

$$DC^2 = 9 - \frac{144}{25} = \frac{81}{25}$$

$$DC = \frac{9}{5}$$

Therefore

$$AD = AC - DC$$

$$= 5 - \frac{9}{5}$$

$$= \frac{16}{5}$$

(i)

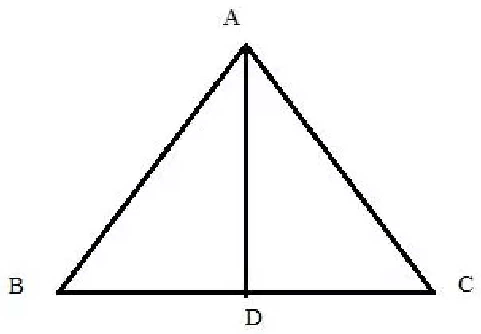
$$\tan \angle DBC = \frac{\text{perpendicular}}{\text{base}} = \frac{DC}{BD} = \frac{\frac{9}{5}}{\frac{12}{5}} = \frac{3}{4}$$

(ii)

$$\sin \angle DBA = \frac{AD}{AB} = \frac{\frac{16}{5}}{4} = \frac{4}{5}$$

Solution 5:

Consider the figure below

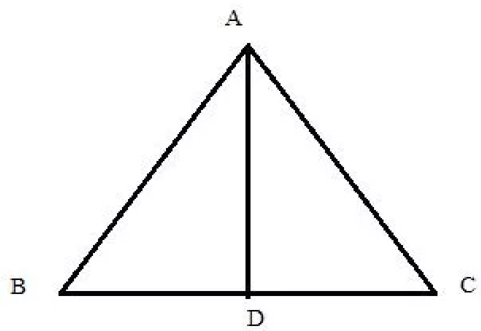


In the isosceles ΔABC, $AB = AC = 15\text{cm}$ and $BC = 18\text{cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9\text{ cm}$

$\cos\angle ABC = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{BD}{AB} = \dfrac{9}{15} = \dfrac{3}{5}$

Solution 6:

Consider the figure below



In the isosceles ΔABC, $AB = AC = 5\text{cm}$ and $BC = 8\text{cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 4\text{ cm}$

Since $\angle ADB = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2\ (AB \text{ is hypotenuse in } \Delta ABD)$

$\Rightarrow AD^2 = 5^2 - 4^2$

$\therefore\ AD^2 = 9$ and $AD = 3$

(i)

$\sin B = \frac{AD}{AB} = \frac{3}{5}$

(ii)

$\tan C = \frac{AD}{DC} = \frac{3}{4}$

(iii)

$\sin B = \frac{AD}{AB} = \frac{3}{5}$

$\cos B = \frac{BD}{AB} = \frac{4}{5}$

Therefore

$\sin^2 B + \cos^2 B$

$= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$

$= \frac{25}{25}$

$= 1$

(iv)

$\tan C = \frac{AD}{DC} = \frac{3}{4}$

$\cot B = \frac{BD}{AD} = \frac{4}{3}$

Therefore

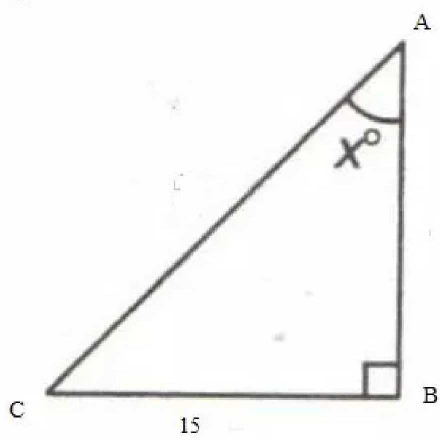
$\tan C - \cot B$

$= \frac{3}{4} - \frac{4}{3}$

$= -\frac{7}{12}$

Solution 7:

Consider the figure



$\tan x^\circ = \frac{3}{4}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{3}{4}$

Therefore if length of base = 4x, length of perpendicular = 3x

Since

$BC^2 + AB^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x$

Now

$BC = 15$

$3x = 15$

$x = 5$

Therefore

$AB = 4x$

$= 4 \times 5$

$= 20\text{cm}$

And

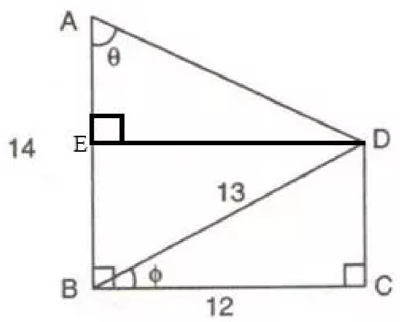
$AC = 5x$

$= 5 \times 5$

$= 25\text{cm}$

Solution 8:

Consider the figure



A perpendicular is drawn from D to the side AB at point E which makes BCDE is a rectangle.
Now in right angled triangle BCD using Pythagorean Theorem

$\Rightarrow BD^2 = BC^2 + CD^2 \ (AB \text{ is hypotenuse in } \Delta ABD)$

$\Rightarrow CD^2 = 13^2 - 12^2 = 25$

$\therefore CD = 5$

Since BCDE is rectangle so ED 12 cm, EB = 5 and AE = 14 - 5 = 9

(i)

$\sin \phi = \frac{CD}{BD} = \frac{5}{13}$

$\tan \theta = \frac{ED}{AE} = \frac{12}{9} = \frac{4}{3}$

(ii)

$\sec \theta = \frac{AD}{AE}$

$\sec \theta = \frac{AD}{9}$

$AD = 9 \sec \theta$

Or

$\text{cosec} \theta = \frac{AD}{ED}$

$\text{cosec} \theta = \frac{AD}{12}$

$AD = 12 \text{cosec} \theta$

Solution 9:

Given

$$\sin B = \frac{4}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5}$$

Therefore if length of perpendicular = 4x, length of hypotenuse = 5x

Since

$$BC^2 + AC^2 = AB^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (4x)^2 = BC^2$$

$$BC^2 = 9x^2$$

$$\therefore BC = 3x$$

Now

$$BC = 15$$

$$3x = 15$$

$$x = 5$$

(i)

$$AC = 4x$$

$$= 4 \times 5$$

$$= 20 \text{ cm}$$

And

$$AB = 5x$$

$$= 5 \times 5$$

$$= 25 \text{cm}$$

(ii)

Given

$$\tan \angle ADC = \frac{1}{1}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{AC}{CD} = \frac{1}{1}$$

Therefore if length of perpendicular = x, length of hypotenuse = x

Since

$$AC^2 + CD^2 = AD^2 \quad [\text{Using Pythagoras Theorem}]$$
$$(x)^2 + (x)^2 = AD^2$$
$$AD^2 = 2x^2$$
$$\therefore AD = \sqrt{2}x$$

Now

$$AC = 20$$
$$x = 20$$

So

$$AD = \sqrt{2}x$$
$$= \sqrt{2} \times 20$$
$$= 20\sqrt{2}\text{ cm}$$

And

$$CD = 20\text{ cm}$$

Now

$$\tan B = \frac{AC}{BC} = \frac{20}{15} = \frac{4}{3}$$
$$\cos B = \frac{BC}{AB} = \frac{15}{25} = \frac{3}{5}$$

So

$$\tan^2 B - \frac{1}{\cos^2 B}$$
$$= \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$$
$$= \frac{16}{9} - \frac{25}{9}$$
$$= -\frac{9}{9}$$
$$= -1$$

Solution 10:

$$\sin A + \text{cosec} A = 2$$

Squaring both sides

$$(\sin A + \text{cosec} A)^2 = 2^2$$
$$\sin^2 A + \text{cosec}^2 A + 2\sin A \cdot \text{cosec} A = 4$$
$$\sin^2 A + \text{cosec}^2 A + 2\cancel{\sin A} \cdot \frac{1}{\cancel{\sin A}} = 4$$
$$\sin^2 A + \text{cosec}^2 A = 2$$

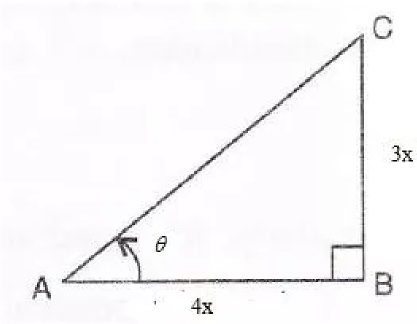
Solution 11:

$$\tan A + \cot A = 5$$

Squaring both sides

$$(\tan A + \cot A)^2 = 5^2$$
$$\tan^2 A + \cot^2 A + 2\tan A \cdot \cot A = 25$$
$$\tan^2 A + \cot^2 A + 2\cancel{\tan A} \cdot \frac{1}{\cancel{\tan A}} = 25$$
$$\tan^2 A + \cot^2 A = 23$$

Consider the diagram below:



$4\sin\theta = 3\cos\theta$

$\tan\theta = \frac{3}{4}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{3}{4} \Rightarrow \frac{BC}{AB} = \frac{3}{4}$

Therefore if length of BC = 3x, length of AB = 4x

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(4x)^2 + (3x)^2 = AC^2$

$AC^2 = 25x^2$

$\therefore AC = 5x(\text{hypotenuse})$

(i)

$\sin\theta = \frac{BC}{AC} = \frac{3}{5}$

(ii)

$\cos\theta = \frac{AB}{AC} = \frac{4}{5}$

Solution 12:

(iii)

$\cot\theta = \frac{AB}{BC} = \frac{4}{3}$

$\text{cosec}\,\theta = \frac{AC}{BC} = \frac{5}{3}$

Therefore

$\cot^2\theta - \text{cosec}^2\theta$

$= \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$

$= \frac{16-25}{9}$

$= -\frac{9}{9}$

$= -1$

(iv)

$4\cos^2\theta - 3\sin^2\theta + 2$

$= 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$

$= \frac{64}{25} - \frac{27}{25} + 2$

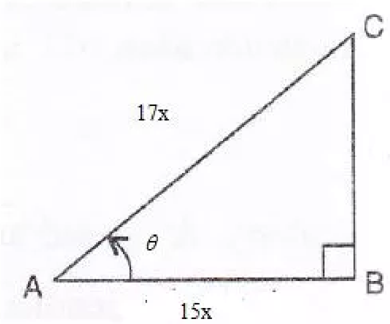
$= \frac{64-27+50}{25}$

$= \frac{87}{25}$

$= 3\frac{12}{25}$

Solution 13:

Consider the diagram below:



$17\cos\theta = 15$

$\cos\theta = \frac{15}{17}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{15}{17} \Rightarrow \frac{AB}{AC} = \frac{15}{17}$

Therefore if length of AB = 15x, length of AC = 17x

Since

$AB^2 + BC^2 = AC^2 \qquad [\text{Using Pythagoras Theorem}]$

$(17x)^2 - (15x)^2 = BC^2$

$BC^2 = 64x^2$

$\therefore\ BC = 8x(\text{perpendicular})$

Now

$\sec\theta = \frac{AC}{AB} = \frac{17}{15}$

$\tan\theta = \frac{BC}{AB} = \frac{8}{15}$

Therefore

$\tan\theta + 2\sec\theta$

$= \frac{8}{15} + 2\cdot\frac{17}{15}$

$= \frac{42}{15}$

$= \frac{14}{5}$

$= 2\frac{4}{5}$

Solution 14:

$5\cos A - 12\sin A = 0$

$5\cos A = 12\sin A$

$\frac{\sin A}{\cos A} = \frac{5}{12}$

$\tan A = \frac{5}{12}$

Now

$\frac{\sin A + \cos A}{2\cos A - \sin A} = \frac{\frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}}{\frac{2\cos A}{\cos A} - \frac{\sin A}{\cos A}}$

$= \frac{\tan A + 1}{2 - \tan A}$

$= \frac{\frac{5}{12} + 1}{2 - \frac{5}{12}}$

$= \frac{\frac{17}{12}}{\frac{19}{12}}$

$= \frac{17}{19}$

Solution 15:

Since D is mid-point of AC so $AC = 2DC$

(i)

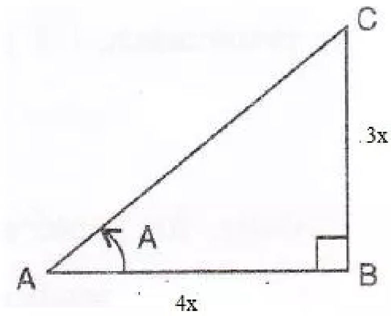
$$\frac{\tan \angle CAB}{\tan \angle CDB}$$
$$= \frac{\frac{BC}{AC}}{\frac{BC}{DC}}$$
$$= \frac{BC}{2DC} \cdot \frac{DC}{BC}$$
$$= \frac{1}{2}$$

(ii)

$$\frac{\tan \angle ABC}{\tan \angle DBC}$$
$$= \frac{\frac{AC}{BC}}{\frac{DC}{BC}}$$
$$= \frac{2DC}{BC} \cdot \frac{BC}{DC}$$
$$= 2$$

Solution 16:

Consider the diagram below:



$$3\cos A = 4\sin A$$
$$\cot A = \frac{4}{3}$$
$$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Therefore if length of AB = 4x, length of BC = 3x

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$
$$(4x)^2 + (3x)^2 = AC^2$$
$$AC^2 = 25x^2$$
$$\therefore AC = 5x(\text{hypotenuse})$$

(i)

$$\cos A = \frac{AB}{AC} = \frac{4}{5}$$

(ii)

$$\text{cosec} A = \frac{AC}{BC} = \frac{5}{3}$$

Therefore

$3-\cot^2 A+\csc^2 A$

$=3-\left(\frac{4}{3}\right)^2+\left(\frac{5}{3}\right)^2$

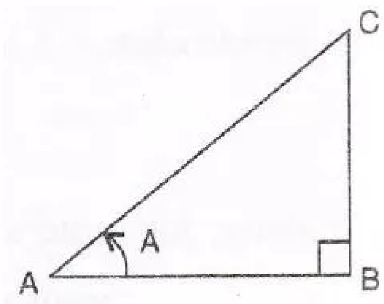
$=\frac{27-16+25}{9}$

$=\frac{36}{9}$

$=4$

Solution 17:

Consider the figure



$\tan A=\frac{75}{100}=\frac{3}{4}$

$i.e. \frac{\text{perpendicular}}{\text{base}}=\frac{BC}{AB}=\frac{3}{4}$

Therefore if length of base = 4x, length of perpendicular = 3x

Since

$BC^2+AB^2=AC^2$ [Using Pythagoras Theorem]

$(3x)^2+(4x)^2=AC^2$

$AC^2=9x^2+16x^2=25x^2$

$\therefore AC=5x$

Now

$AC=30$

$5x=30$

$x=6$

Therefore

$AB=4x$

$=4\times 6$

$=24\text{cm}$

And

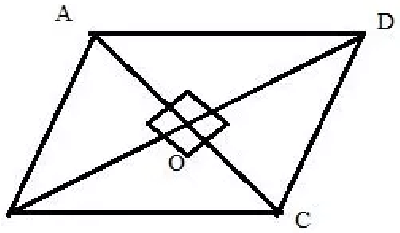
$BC=3x$

$=3\times 6$

$=18\text{cm}$

Solution 18:

Consider the figure



The diagonals of a rhombus bisects each other perpendicularly

$\cos\angle CAB = \frac{6}{10} = \frac{3}{5}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{OA}{AB} = \frac{3}{5}$

Therefore if length of base = 3x, length of hypotenuse = 5x

Since

$OB^2 + OA^2 = AB^2 \quad [\text{Using Pythagoras Theorem}]$

$(5x)^2 - (3x)^2 = OB^2$

$OB^2 = 16x^2$

$\therefore OB = 4x$

Now

$OB = 8$

$4x = 8$

$x = 2$

Therefore

$AB = 5x$

$= 5\times 2$

$= 10\,\text{cm}$

And

$OA = 3x$

$= 3\times 2$

$= 6\,\text{cm}$

Since the sides of a rhombus are equal so the length of the side of the rhombus $= 10\,\text{cm}$

The diagonals are

$BD = 8\times 2$

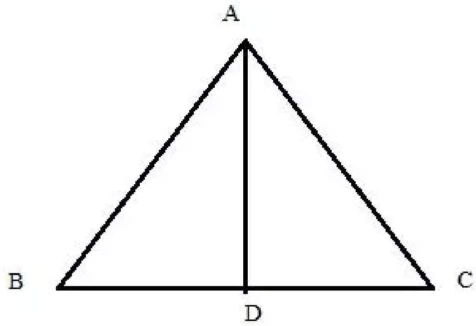
$= 16\,\text{cm}$

$AC = 6\times 2$

$= 12\,\text{cm}$

Solution 19:

Consider the figure below



In the isosceles ΔABC, the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9\,\text{cm}$

Since $\angle ADB = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2$ (AB is hypotenuse in ΔABD)

$\Rightarrow AD^2 = 15^2 - 9^2$

$\therefore \quad AD^2 = 144$ and $AD = 12$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{9}{15} = \frac{3}{5}$$

(ii)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{15} = \frac{4}{5}$$

(iii)

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{9} = \frac{4}{3}$$

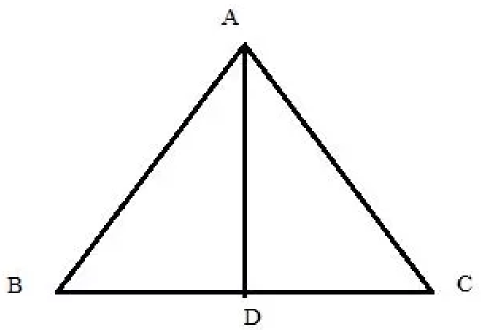
$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{15}{9} = \frac{5}{3}$$

Therefore

$$\tan^2 B - \sec^2 B + 2$$
$$= \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2 + 2$$
$$= \frac{16 - 25 + 18}{9}$$
$$= \frac{9}{9}$$
$$= 1$$

Solution 20:

Consider the figure below



$$\sin B = \frac{8}{10} = \frac{4}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{4}{5}$$

Therefore if length of perpendicular = 4x, length of hypotenuse = 5x

Since

$AD^2 + BD^2 = AB^2$ [Using Pythagoras Theorem]

$(5x)^2 - (4x)^2 = BD^2$

$BD^2 = 9x^2$

$\therefore BD = 3x$

Now

$BD = 9$

$3x = 9$

$x = 3$

Therefore

$$AB = 5x$$
$$= 5 \times 3$$
$$= 15\,\text{cm}$$

And

$$AD = 4x$$
$$= 4 \times 3$$
$$= 12\,\text{cm}$$

Again

$$\tan C = \frac{1}{1}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{1}{1}$$

Therefore if length of perpendicular = x, length of base = x

Since

$$AD^2 + DC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$
$$(x)^2 + (x)^2 = AC^2$$
$$AC^2 = 2x^2$$
$$\therefore AC = \sqrt{2}x$$

Now

$$AD = 12$$
$$x = 12$$

Therefore

$$DC = x$$
$$= 12\,\text{cm}$$

And

$$AC = \sqrt{2}x$$
$$= \sqrt{2} \times 12$$
$$= 12\sqrt{2}\,\text{cm}$$

Solution 21:

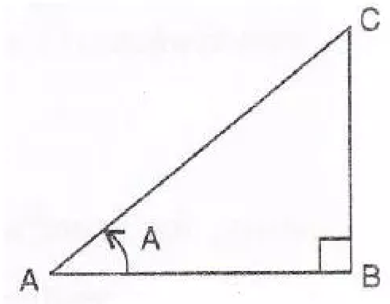
$$q \tan A = p$$
$$\tan A = \frac{p}{q}$$

Now

$$\frac{p \sin A - q \cos A}{p \sin A + q \cos A} = \frac{\frac{p \sin A}{\cos A} - \frac{q \cos A}{\cos A}}{\frac{p \sin A}{\cos A} + \frac{q \cos A}{\cos A}}$$
$$= \frac{p \tan A - q}{p \tan A + q}$$
$$= \frac{p\left(\frac{p}{q}\right) - q}{p\left(\frac{p}{q}\right) + q}$$
$$= \frac{\frac{p^2 - q^2}{q}}{\frac{p^2 + q^2}{q}}$$
$$= \frac{p^2 - q^2}{p^2 + q^2}$$

Solution 22:

Consider the figure



$\sin A = \cos A$

$\tan A = \frac{1}{1}$

$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{1}{1}$

Therefore if length of perpendicular = x, length of base = x

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(x)^2 + (x)^2 = AC^2$

$AC^2 = 2x^2$

$\therefore AC = \sqrt{2}x$

Now

$\sec A = \frac{AC}{AB} = \sqrt{2}$

Therefore

$2\tan^2 A - 2\sec^2 A + 5$

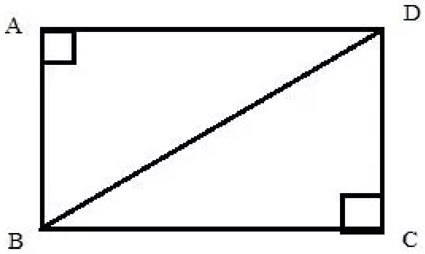
$= 2(1)^2 - 2(\sqrt{2})^2 + 5$

$= 2 - 4 + 5$

$= 3$

Solution 23:

Consider the diagram



$$\cot\angle ABD = \frac{15}{10} = \frac{3}{2}$$

$$i.e.\ \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{3}{2}$$

Therefore if length of base = 3x, length of perpendicular = 2x

Since

$$AB^2 + AD^2 = BD^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (2x)^2 = BD^2$$

$$BD^2 = 13x^2$$

$$\therefore\ BD = \sqrt{13}x$$

Now

$$BD = 26$$

$$\sqrt{13}x = 26$$

$$x = \frac{26}{\sqrt{13}}$$

Therefore

$$AD = 2x$$

$$= 2\times\frac{26}{\sqrt{13}}$$

$$= \frac{52}{\sqrt{13}}\text{cm}$$

$$AB = 3x$$

$$= 3\times\frac{26}{\sqrt{13}}$$

$$= \frac{78}{\sqrt{13}}\text{cm}$$

Now

$$\text{Area of rectangle } ABCD = AB\times AD$$

$$= \frac{78}{\sqrt{13}}\times\frac{52}{\sqrt{13}}$$

$$= 312\text{ cm}^2$$

$$\text{Perimeter of rectangle } ABCD = 2(AB + AD)$$

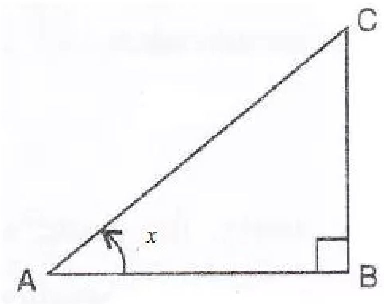
$$= 2\left(\frac{78}{\sqrt{13}} + \frac{52}{\sqrt{13}}\right)$$

$$= \frac{260}{\sqrt{13}}$$

$$= 20\sqrt{13}\text{ cm}$$

Solution 24:

Consider the figure



$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Therefore if length of $\text{perpendicular} = \sqrt{3}x$, length of $\text{hypotenuse} = 2x$

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(2x)^2 - (\sqrt{3}x)^2 = AB^2$

$AB^2 = x^2$

$\therefore AB = x$

Now

$\cos x = \frac{AB}{AC} = \frac{1}{2}$

(i)

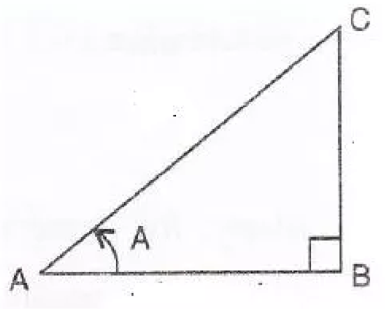
$$4\sin^3 x - 3\sin x = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$
$$= 0$$

(ii)

$$3\cos x - 4\cos^3 x = 3\cdot\frac{1}{2} - 4\cdot\left(\frac{1}{2}\right)^3$$
$$= \frac{3}{2} - \frac{1}{2}$$
$$= 1$$

Solution 25:

Consider the diagram below:



$\sin A = \frac{\sqrt{3}}{2}$

$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Therefore if length of $BC = \sqrt{3}x$, length of $AC = 2x$

Since

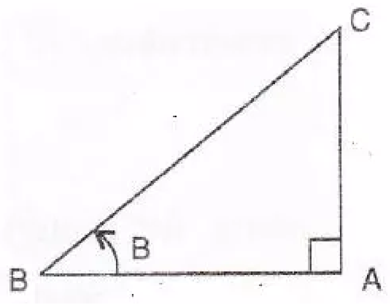
$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$\left(\sqrt{3}x\right)^2 + AB^2 = (2x)^2$

$AB^2 = x^2$

$\therefore AB = x(\text{base})$

Consider the diagram below:



$\cos B = \frac{\sqrt{3}}{2}$

$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{3}}{2}$

Therefore if length of $AB = \sqrt{3}x$, length of $BC = 2x$

Since

$AB^2 + AC^2 = BC^2$ [Using Pythagoras Theorem]

$AC^2 + \left(\sqrt{3}x\right)^2 = (2x)^2$

$AC^2 = x^2$

$\therefore AC = x(\text{perpendicular})$

Now

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$

$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$

Therefore

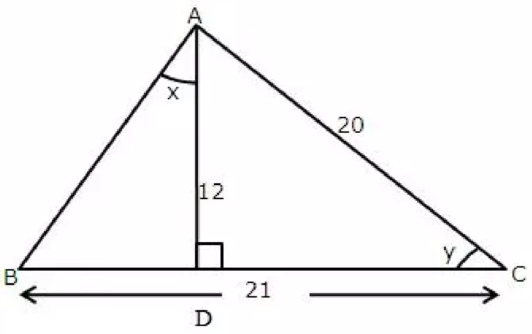
$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

Solution 26:

Consider the given diagram as



Using Pythagorean Theorem

$$AD^2 + DC^2 = AC^2$$
$$DC^2 = 20^2 - 12^2 = 256$$
$$DC = 16$$

Now

$$BC = BD + DC$$
$$21 = BD + 16$$
$$BD = 5$$

Again using Pythagorean Theorem

$$AD^2 + BD^2 = AB^2$$
$$12^2 + 5^2 = AB^2$$
$$AB^2 = 169$$
$$AB = 13$$

Now

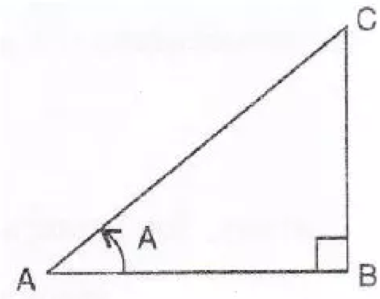
$$\sin x = \frac{BD}{AB} = \frac{5}{13}$$
$$\sin y = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$
$$\cot y = \frac{DC}{AD} = \frac{16}{12} = \frac{4}{3}$$

Therefore

$$\frac{10}{\sin x} + \frac{6}{\sin y} - 6\cot y = \frac{10}{\frac{5}{13}} + \frac{6}{\frac{3}{5}} - 6\left(\frac{4}{3}\right)$$
$$= \frac{130}{5} + \frac{30}{3} - \frac{24}{3}$$
$$= 26 + 10 - 8$$
$$= 28$$

Solution 27:

Consider the figure



$\sec A = \sqrt{2}$

$i.e. \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \sqrt{2}$

Therefore if length of $\text{base} = x$, length of $\text{hypotenuse} = \sqrt{2}x$

Since

$AB^2 + BC^2 = AC^2$ $\quad$ [Using Pythagoras Theorem]

$\left(\sqrt{2}x\right)^2 - (x)^2 = BC^2$

$BC^2 = x^2$

$\therefore BC = x$

Now

$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{2}}$

$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$

$\tan A = \frac{BC}{AB} = 1$

$\cot A = \frac{1}{\tan A} = 1$

Therefore

$$\frac{3\cot^2 A + 2\sin^2 A}{\tan^2 A - \cos^2 A} = \frac{3(1)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2}{1^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{3+1}{1-\frac{1}{2}}$$

$$= \frac{4}{\frac{1}{2}}$$

$$= 8$$

Solution 28:

$\cos\theta = \frac{3}{5}$

Now

$$\frac{\text{cosec}\,\theta - \cot\theta}{\text{cosec}\,\theta + \cot\theta} = \frac{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\frac{1-\cos\theta}{\sin\theta}}{\frac{1+\cos\theta}{\sin\theta}}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\frac{3}{5}}{1+\frac{3}{5}}$$

$$= \frac{\frac{2}{5}}{\frac{8}{5}}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

Solution 29:

$\text{cosec}A + \sin A = 5\frac{1}{5}$

Squaring both sides

$$(\text{cosec}A + \sin A)^2 = \left(5\frac{1}{5}\right)^2$$

$$\text{cosec}^2 A + \sin^2 A + 2\,\text{cosec}A \cdot \frac{1}{\text{cosec}A} = \frac{26}{25}$$

$$\text{cosec}^2 A + \sin^2 A = \frac{626}{25}$$

$$\text{cosec}^2 A + \sin^2 A = 25\frac{1}{25}$$

Solution 30:

$5\cos\theta = 6\sin\theta$

$\tan\theta = \frac{5}{6}$

Now

(i)

$\tan\theta = \frac{5}{6}$

(ii)

$$\frac{12\sin\theta - 3\cos\theta}{12\sin\theta + 3\cos\theta} = \frac{\frac{12\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{12\sin\theta}{\cos\theta} + \frac{3\cos\theta}{\cos\theta}}$$

$$= \frac{12\tan\theta - 3}{12\tan\theta + 3}$$

$$= \frac{12\left(\frac{5}{6}\right) - 3}{12\left(\frac{5}{6}\right) + 3}$$

$$= \frac{\frac{42}{6}}{\frac{78}{6}}$$

$$= \frac{42}{78}$$

$$= \frac{7}{13}$$